Structural mechanics
[A probabilistic study of nonlinear behavior in beams resting on tensionless soil with geometric considerations]

Seguini Meriem[1*], Nedjar Djamel[1]

[1]University of Science and Technology of Oran Mohamed Boudiaf, Oran, Algeria
[*]Corresponding author: meriem.seguini@univ-usto.dz

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ABSTRACT

The nonlinear analysis of a beam resting on nonlinear random tensionless soil was studied with the aim of quantifying the influence of the spatial variability of the tension soil characteristics on the behavior of the beam and illustrating the importance of the geometric nonlinear analysis of a beam. The soil-structure interaction mechanism is taken into account where the soil is modeled as nonlinear. Due to large deflections and moderate rotations of the beam, the Von-Kàrman type nonlinearity based on the finite element formulation of nonlinear beam response is adopted, and the frictional resistance at the beam's interface is taken into account. The study assesses the impact of various factors, including geometric and material nonlinearities, as well as the spatial variability of soil properties, aiming to understand the behavior of the beam in real-world conditions. Additionally, the study seeks to determine the effectiveness of a probabilistic approach in evaluating the reliability of the beam's response. The results indicate that the probabilistic approach of the soil and the geometric nonlinearity of the beam serve a major role in the evaluations of the beam response.

1. Introduction

For many decades, the beam was often considered the simplest element and has been extensively used to model various mechanical structures. However, the behavior of complex structures composed of beam elements, characterized by strong nonlinearity, can be significantly influenced by the mechanism of soil-structure interaction.

Furthermore, the study of the linear and nonlinear geometric behavior of the beam, based on various theories, and its response to static and dynamic loads have been developed (Irschik & Gerstmayr, 2009; Reddy, 2015; Santos, 2015).

To investigate the nonlinear geometric behavior of the buried structures resting on both linear and nonlinear soil within the context of soil-structure interaction, other models have been developed. Kordkheili and Bahai (2008); Kordkheili, Bahai, and Mirtaheri (2011) studied the geometric nonlinear static analysis of the pipe and dynamic analysis of 3D flexible riser structures. (Jang, 2013) presented a semi-analytical approach for analyzing geometrically nonlinear beams resting on linear elastic foundations, using the von Kàrmàn Euler-Bernoulli theory.

Some research has focused on the study of the material nonlinear behavior of the beams resting on both linear elastic and nonlinear elastic-plastic soil. To overcome this problem, several analytical methods have been proposed. Ayoub (2003) developed a new mixed formulation for the analysis of an inelastic beam resting on an elastic-plastic nonlinear foundation. Additionally, Mullapudi and Ayoub (2010) extended the same work initially developed by Ayoub (2003) to study the inelastic response of a beam resting on two-parameter foundations where the analysis is based on the Vlasov and Pasternak approach. Sapountzakis and Kampitsis (2011) introduced the boundary element method to analyze the nonlinear dynamic behavior of moderately large deflections in a beam column resting on a tensionless Winkler foundation. Moreover, Sapountzakis and Kampitsis (2013) used the boundary element method to analyze the nonlinear behavior of an inelastic Euler-Bernoulli beam resting on a nonlinear two-parameter elastoplasic foundation and compared the results of the study with those obtained by Ayoub (2003) and Mullapudi and Ayoub (2010).

Other researchers have focused on the probabilistic analysis of different geotechnical structures (Fenton & Griffiths, 2003; Lacasse, 2001; Phoon & Kulhawy, 1999; Popescu, Deodatis, & Nobahar, 2005; VanMarcke, 1983). In these studies, heterogeneous soil is considered, characterized by its spatial properties such as the mean, coefficient of variation, and correlation length (Fenton & Vanmarcke, 1990). Furthermore, recent research on soil-structure interaction including stochastic static and dynamic analysis of many structures, has been conducted. Griffiths, Paiboon, Huang, and Fenton (2008) employed a probabilistic approach to analyze the behavior of laterally loaded piles supported by random Winkler soil. The linear dynamic response of pipes resting on random soil has also been studied (Nedjar, Hamane, Bensafi, Elachachi, & Breysse, 2007; Seguini & Nedjar, 2020). Additionally, various models have been developed to assess the probabilistic behavior of pipes. Seguini and Nedjar (2017a) analyzed the nonlinear behavior of buried pipes subjected to a distributed load and resting on linear random soil, using the finite element and difference methods combined with VanMarcke's theory. Moreover, Seguini and Nedjar (2017b) combined the geometric nonlinearity of the beam with the nonlinearity of the soil to determine the real behavior of a beam. The Neural Network method (ANN) has also been utilized to analyze the effect of the variation in the coefficient of subgrade reaction on the displacement of pipes (Seguini, Khatir, Nedjar, & Wahab, 2022).

In this paper, a finite element numerical model for the geometrically and material nonlinear analysis of a beam resting on nonlinear random soil has been developed. A probabilistic approach was adopted to quantify the effect of soil characteristics on the beam's response. The geometric nonlinear response of the elastoplastic beam is obtained using the von Kàrmàn formulation, based on the assumption of large deflection and moderate rotations of the beam. The tensile force along the beam is assumed to be constant, and the soil is modeled as a random field.

2. Theory and formulation

2.1. Deterministic case

Let's consider a one-dimensional beam of length L, resting on the Winkler foundation, having the origin O where b and h_b represent the width and height of the beam as shown in Figure 1. The beam is subjected to a concentrated load q and is divided into several elements I, each of length l_i.

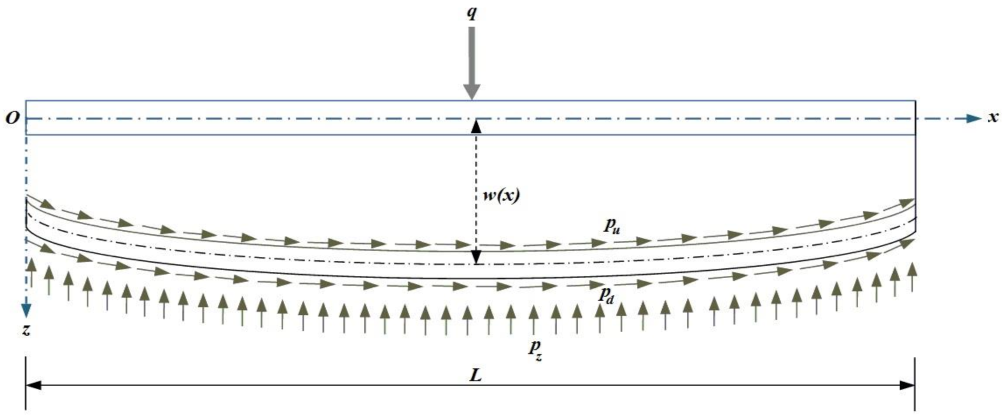

Figure 1. Model of the tensionless beam resting on the foundation and subjected to concentrated load

In this study, it is assumed that:

1. The beam-soil interaction system is represented by the horizontal linear and vertical nonlinear springs.

2. The tensionless nonlinear soil is represented by the elastic-plastic force deformation relation.

3. The geometric and materially nonlinear behavior of the beam is analyzed using the Euler-Bernoulli von Kàrmàn theory and the elastic-plastic force deformation relation.

Based on the Euler-Bernoulli theory, the equation obtained from the applied equilibrium forces and moments of the deformed beam element in x and z directions can be written as follows (see Figure 2):

$$dN_i = (p_{ux,i} + p_{dx,i})dx_i \tag{1}$$

$$dM_i = \left[Q_i - \frac{h_b}{2}(p_{ux,i} - p_{dx,i}) - N_i \frac{dw_i}{dx_i}\right] dx_i \tag{2}$$

$$dQ_i = [p_{z,i} - (p_{uz,i} + p_{dz,i})]dx_i \tag{3}$$

Where N is the tensile force, M is the bending moment and w is the beam's vertical displacement.

p_{ux} and p_{dx} represent the interfacial shearing resistance forces at the top and the bottom of the beam in the x-direction, respectively. p_{uz} and p_{dz} denote the vertical subgrade reaction of the soil in the z-direction. These forces can be expressed as follows:

$$\begin{cases} p_{ux,i} = k_{ux,i} u_i^u \\ p_{dx,i} = k_{dx,i} u_i^d \end{cases} \tag{4}$$

$$\begin{cases} p_{uz,i} = k_{uz,i}.tan(-\theta_i) \\ p_{dz,i} = k_{dz,i}.tan(-\theta_i) \end{cases} \tag{5}$$

u^u and u^d represent the horizontal displacements, while k_{ux} and k_{dx} are the horizontal coefficients of the soil's subgrade reaction. These coefficients can be determined through experimental tests or empirical methods (Bowles, 1988), k_{uz} and k_{dz} are the nonlinear vertical coefficients of the soil's subgrade reaction. Assuming that the rotation of the beam is small, *tan* $(-\theta_i) = (-\theta_i)$, the Equation (3) can be expressed as:

$$dQ_i = [p_{z,i} - (k_{uz,i} + k_{dz,i})\theta_i]dx_i \tag{6}$$

$$\text{with} \qquad p_{z,i} = k_i.w_i \tag{7}$$

The actual tensile force N_i of the beam is given by

$$N_i = \int_A \sigma_x \, dA = \int_A E(\varepsilon_x^{el} + \varepsilon_x^{pl})dA = EA.(u_i^0)' + \frac{\chi}{2}EA.(w_i')^2 \tag{8}$$

$$\text{with} \qquad \begin{cases} u_i^u = u_i^0 - \frac{h_b}{2}w_i' \\ u_i^d = u_i^0 + \frac{h_b}{2}w_i' \end{cases} \tag{9}$$

Where σ_x, ε_x^{el}, and ε_x^{pl} are the stress, the elastic, and the plastic strain, respectively. u_i^u and u_i^d is the horizontal displacement of the beam. χ is a nonlinear coefficient.

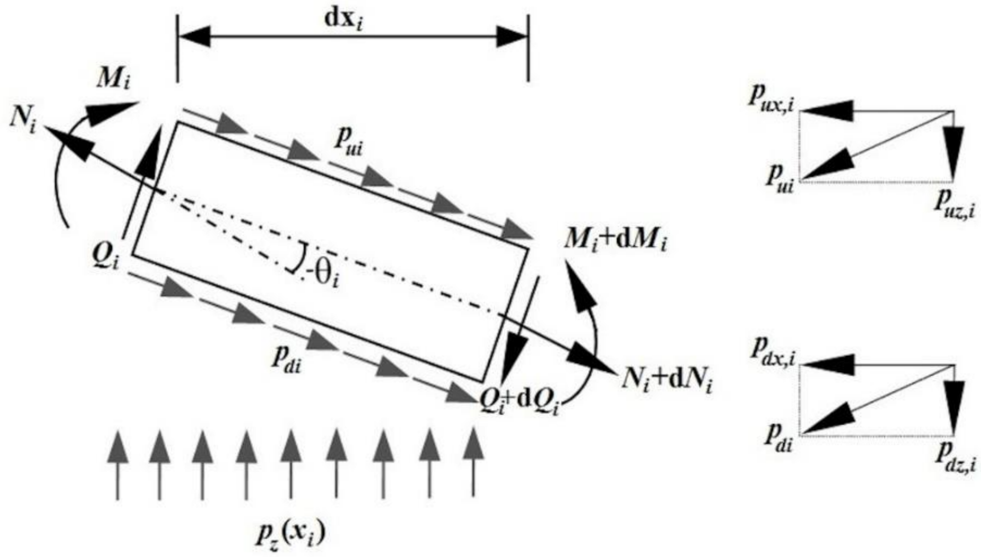

Figure 2. Nonlinear deformed beam element showing displacement and forces

Therefore, by substituting the Equation (3) and (7) and the relationship $M = -EI\frac{d^2w_i}{dx_i^2}$, $\theta_i = \frac{dw_i}{dx_i}$ into Eq.(2) which differentiating it against x_i another time, the equation of the beam-soil interaction system can be obtained as follows:

$$EI\frac{d^4w_i}{dx_i^4} - \left[N_i + \frac{(k_{ux}-k_{dx})}{4}h^2\right]\frac{d^2w_i}{dx_i^2} + (k_iw_i) - \frac{h_b}{2}(k_{ux} - k_{dx})(u_i^0)' = q_i \quad (10)$$

In order to solve Equation (10), where q is the external force, the total potential energy functional is used, and the matrix of rigidity of the beam element resting on a nonlinear foundation is obtained as:

$$K_i = K_{bl,i} + K_{bnl,i}\{\Delta_i\}^k + K_{f,i} \quad (11)$$

$$\{\Delta_i\}^{k+1} = \{\Delta_i\}^k + \{\delta\Delta\} \quad (12)$$

$$\{\Delta_i\} = \begin{Bmatrix} u_1 \\ w_1 \\ \theta_1 \\ u_2 \\ w_2 \\ \theta_2 \end{Bmatrix} \quad (13)$$

Where $K_{bl,i}$is the linear matrix of rigidity of the beam, $K_{bnl,i}$ is the nonlinear tangent matrix of the beam, which depends on the vector of the nodal displacement, and $K_{f,i}$ is the nonlinear matrix of rigidity of the soil:

$$K_{bl,i} = \int_0^{li} a_{,xx}^T . k^{i-1}(x). a_{,xx}(x)dx \quad (14)$$

$$K_{bnl,i} = \int_0^{li} N_i + \frac{b(k_{ux}+k_{dx})h^2}{4}. a_{,xx}\, dx - \frac{h}{2}(k_{ux,i} + k_{dx,i})(u_i^0) \quad (15)$$

$$K_{f,i} = \int_0^{l_i} a^T(x)k_f^{i-1}(x)a(x) \quad (16)$$

Where k and k_f are the beam section and foundation stiffness terms, respectively, as defined by Bowles (1988), and $a(x)$ is the vector of the displacement interpolation function.

2.2. Probabilistic case

A probabilistic approach based on the Monte Carlo method is adopted in this study, characterized by the following noteworthy features (Seguini & Nedjar, 2017a):

1. The soil is subdivided into several zones.

2. The properties of the random coefficient of subgrade reaction of soil are defined as follows: its constant mean value m_k, variance σ_k, and correlation length L_c which describes the distance over which the correlation between soil properties tends to disappear.

3. A series of Monte Carlo simulations with *n* realizations are performed in order to obtain a cumulative distribution function.

4. The structural response of the beam is statically analyzed.

However, the spatial varying random soil is generated using the local average theory developed by VanMarcke (1983), which is then combined with the elastic-plastic finite element algorithm using the Monte Carlo and the Newton-Raphson methods. This procedure is implemented in a Matlab program. The subscripts *k*, *j*, and *i* are used to define the iterative cycles, *m* is the load step increment, and δ represents the incremental quantities. Moreover, the following steps of the present analysis are:

Step 1: Using the Matlab software to introduce the deterministic and probabilistic properties of the beam and soil.

Step 2: Discretization of the random soil, generation of the lognormal distribution of random k_z, and evaluation of the displacement vector $\{\Delta^1\}$ of the linear system for k=1, m=1, j=1, and i=1 by using the Equation (11) where it is assumed that the nonlinear rigidity matrices of the beam and the soil are equal to zero. In contrast, if m $\geq$ 1 and j > 1, these matrices are not equal to zero, the global stiffness matrix is calculated, and the displacement vector of the nonlinear system is determined by also using the Equation (11) with $\Delta^{i+1} = \Delta^i + \delta\Delta^i$.

Step 3: Check the global convergence. The equilibrium system is verified through the following inequality:

$$\left|\frac{w_i^j - w_i^{j-1}}{w_i^j}\right| \prec \xi$$
$$\left|\frac{(u_i^0)^j - (u_i^0)^{j-1}}{(u_i^0)^j}\right| \prec \xi \tag{17}$$

Where ξ is an acceptable degree of convergence; in fact, a tolerance of 10^{-3} is adopted in this study. The process represented by step (2) is repeated until the convergence criterion is achieved.

Step 4: Compute the total displacement, stress, and reaction of the soil, and then check the convergence of the soil by using a loop over the soil convergence.

Step 5: Continue with the increments of the external loading until the total loading is undertaken. Then, the result (deflection, tension force, etc.) is computed for *k* simulations. in fact, steps (2)-(4) are repeated until the total number of desired simulations is reached.

3. Methodology

In the numerical study, a geometric and material nonlinear finite element analysis of a beam resting on elastic-plastic soil and subjected to a concentrated load is conducted. The summary of the relevant properties used in the numerical analysis is presented in Table 1. Both deterministic and probabilistic analyses have been carried out, taking into account the characteristics of the tensionless soil, with the aim of obtaining an optimal result for the beam's response. The developed model is evaluated through comparisons between the obtained results and those from the literature (Ayoub, 2003; Sapountzakis & Kampitsis, 2013).

Table 1

Beam and soil properties

Symbol	Parameters	Range of values	Unit
b	Width of beam	6.26	in
h	Height of beam	6.26	in
L	Length of beam	300	in
E_b	Beam Elastic Young modulus	29000	ksi
σ_y	Yield strength of beam	30	ksi
h_b	Strain hardening slope of beam	0.014	(-)
k_u	Horizontal coefficient of soil's reaction at the top of beam	0.75	kip/in^2
k_d	Horizontal coefficient of soil's reaction at the bottom of beam	0.5	kip/in^2
k_z	Vertical coefficient of soil's subgrade reaction	0.5	kip/in^2
q_f	Yield force of soil	1	k/in
h_s	Hardening slope of soil	0.01	(-)
L_c	Correlation length of soil	100,200	in
Cv	Coefficient of variation of soil	30,70	%
q	Applied load	160	kip

Source: The researcher's data analysis

4. Results and discussion

Table 2 presents the convergence study for different numbers of elements (i) by computing the maximum deflection at the midpoint of the beam, according to the nonlinear theory of a beam without taking into account the interfacial resistance. The results are in good agreement with those obtained by Ayoub (2003). In fact, it is observed from this table that the maximum deflection is approximately 0.01% compared to the one obtained by Ayoub (2003) and Sapountzakis and Kampitsis (2013). Moreover, the numerical accuracy is satisfactory when the beam is discretized with a large number of elements (i=12).

Table 2

Maximum vertical deflection w_{max} of a material nonlinear beam resting on nonlinear soil for different numbers of elements i (deterministic analysis)

Without interfacial resistance (Deterministic analysis)				
	Present study		**(Sapountzakis & Kampitsis, 2013)**	**(Ayoub, 2003)**
	(i=8)	**(i=12)**		
W_{max} (in)	3.285	3.971	3.937	3.975

To further define the real behavior of the beam and illustrate the importance of the geometric nonlinear analysis, as well as the effects of interface shearing resistance forces and the spatial variability of soil properties, the large deflection and deformation of the beam resting on tensionless homogeneous and heterogeneous soil have been analyzed. Figure 3 shows the maximum vertical displacement of the beam using two types of analysis (deterministic and probabilistic). From the deterministic analysis, it is evident that when both types of nonlinearities are considered, the maximum deflection decreases by approximately 17%. This significant change is primarily due to the effect of the maximum tensile force within the beam, which decreases by about 13% from 488.41kip to 426.12kip, as observed in Table 3. In fact, when the geometric nonlinearity is taken into account the tensile force increases, making the beam stiffer.

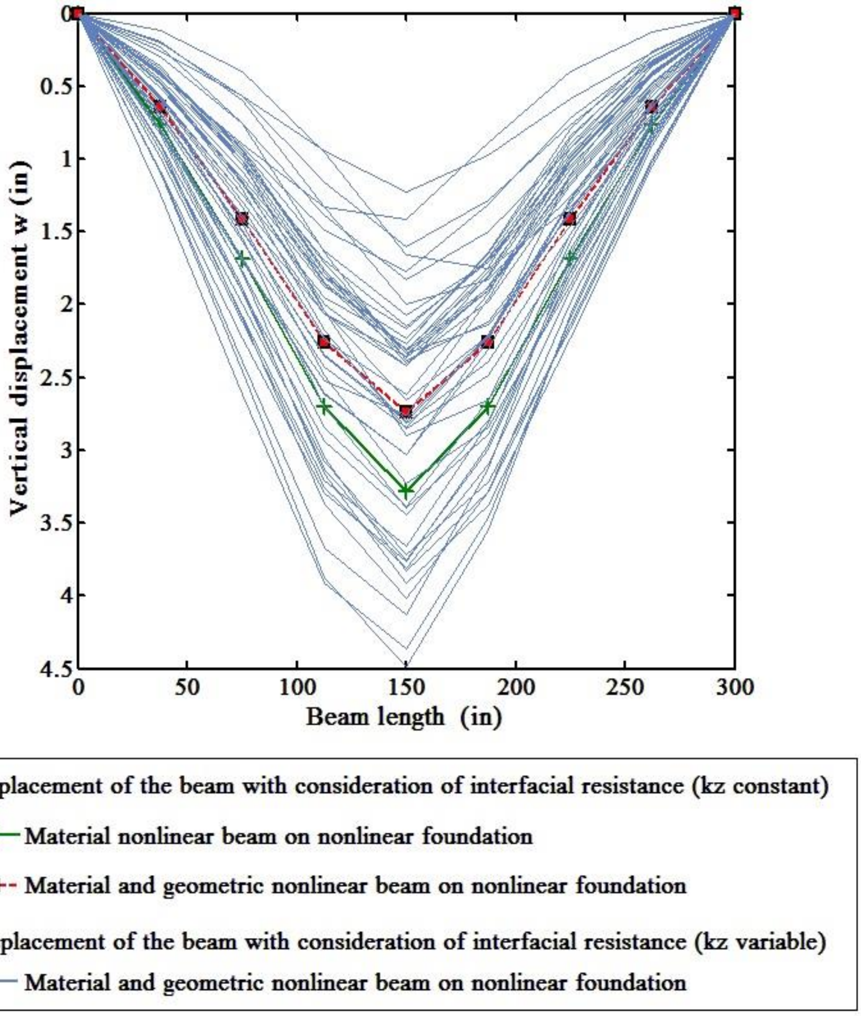

Figure 3. Displacement of the beam under a concentrated load using different types of analysis

To assess the probabilistic behavior of a geometric and material nonlinear beam resting on tensionless random soil, 40 curves of beam displacement has been randomly selected from 1,000 realizations, as shown in Figure 3, and the maximum displacement is mentioned in Table 3. The obtained results indicate that soil heterogeneity tends to increase the displacement and decrease the tensile force of the beam by approximately 27% and 49% respectively, compared to the results obtained in the deterministic analysis.

In summary, the probabilistic approach is essential for addressing uncertainty related to soil variability, while deterministic analysis is more suitable when the soil is considered homogeneous and constant. However, soils are rarely perfectly homogeneous, and the variation of coefficient of soil's subgrade reaction based on the probabilistic analysis can really impact the beam's response.

Table 3

Maximum vertical deflection W_{max} and tension force T_{max}, of geometric and material nonlinear beam resting on nonlinear tensionless (deterministic and probabilistic analysis)

With interfacial resistance			
	Deterministic analysis (K_z = constant)		Probabilistic analysis (k_z = variable)
	Material and geometric nonlinear beam	Material nonlinear beam	Material and geometric nonlinear beam
W_{max} (in)	2.740	3.285	4.484
T_{max} (kip)	488.41	426.12	217.64

Effect of ku and kz on the beam response

As shown in Figure 4, the response of the beam can be also influenced by variation in the coefficients of the soil. It can be observed that when the horizontal and vertical coefficients of the soil decrease, the vertical displacement increases. Furthermore, both coefficients have a significant impact on the beam's response.

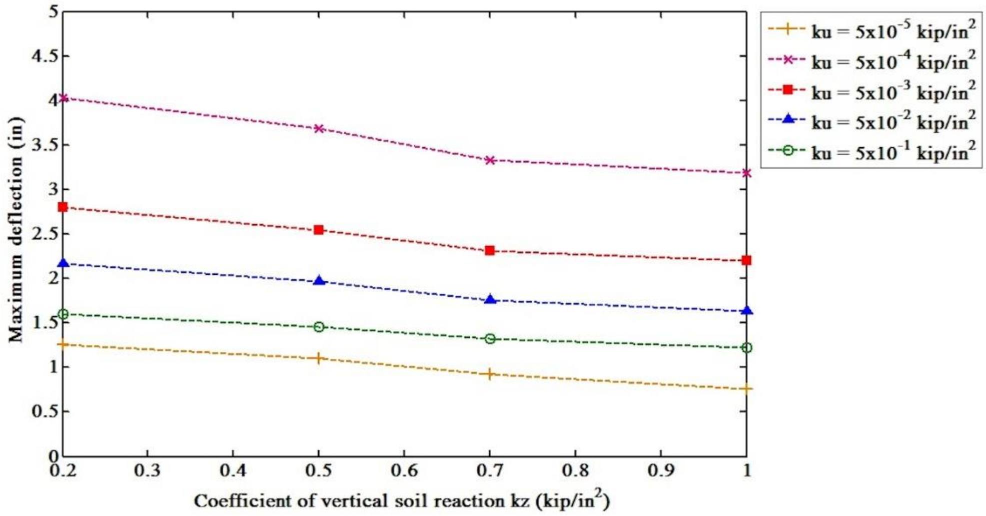

Figure 4. Effect of the variation of horizontal and vertical coefficient of soil's subgrade reaction on the beam response

5. Conclusions

Deterministic and probabilistic nonlinear analyses of a beam resting on a tensionless Winkler foundation and subjected to a concentrated load have been investigated using the Euler-Bernoulli theory for the beam, which undergoes large deflections and deformations. This study considers the influence of several factors, including the geometric nonlinearity of the beam, the shear force along the beam induced by the applied axial reaction of the soil, the material nonlinearity of both the beam and the soil, and the spatial variability of soil properties. The effects of certain soil properties on the response of the beam have been examined in detail. To adequately capture the essential features, the following main conclusions have been drawn:

- In the deterministic case, the analysis demonstrates that the newly proposed model provides more realistic results by considering both the materials and geometric nonlinearities of the beam resting on nonlinear, tensionless soil. When only the material nonlinearity of the beam is considered, the results have been validated through comparison with available literature.

- The probabilistic approach allows for the consideration of this soil variability and the evaluation of the beam's reliability under real conditions. It can help identify potential risk areas and design the structure to be more robust in the face of soil variations. However, the obtained results suggest that the probabilistic approach based on the Monte Carlo method, is a convincing and powerful innovation for estimating the real response of the beam. This approach is essential in determining the actual behavior of the structure.

- The tensionless character and the vertical coefficient of the soil's subgrade reaction have a dominant effect on the deflection of the beam; in fact, the results are sensitive to any changes in the soil's parameters.

- Therefore, this application developed in this paper should be extended by introducing the seismic effect to mitigate such phenomena, thereby improving the design of the beam. This will be of interest to the scientific and engineering community.

References

Ayoub, A. (2003). Mixed formulation of nonlinear beam on foundation elements. *Comptures & Structures, 81*(7), 411-421.

Bowles, J. (1988). *Foundation analysis and design*. New York, NY: McGraw-Hill, Inc.

Fenton, G. A., & Griffiths, D. V. (2003). Bearing-capacity prediction of spatially random c φ soils. *Canadian Geotechnical Journal, 40*(1), 54-65.

Fenton, G. A., & Vanmarcke, E. H. (1990). Simulation of random fields via local average subdivision. *Journal of Engineering Mechanics, 116*(8), 1733-1749.

Griffiths, D., Paiboon, J., Huang, J., & Fenton, G. A. (2008). *Numerical analysis of beams on random elastic foundations.* Paper presented at the Proceedings of the 9th international congress on numerical methods in engineering and scientific applications, CIMENICS.

Irschik, H., & Gerstmayr, J. (2009). A continuum mechanics based derivation of Reissner's large-displacement finite-strain beam theory: The case of plane deformations of originally straight Bernoulli-Euler beams. *Acta Mechanica, 206*(1/2), 1-21.

Jang, T. S. (2013). A new semi-analytical approach to large deflections of Bernoulli–Euler-v. Karman beams on a linear elastic foundation: Nonlinear analysis of infinite beams. *International Journal of Mechanical Sciences, 66*, 22-32.

Kordkheili, S. H., & Bahai, H. (2008). Non-linear finite element analysis of flexible risers in presence of buoyancy force and seabed interaction boundary condition. *Archive of Applied Mechanics, 78*, 765-774.

Kordkheili, S. H., Bahai, H., & Mirtaheri, M. M. (2011). An updated Lagrangian finite element formulation for large displacement dynamic analysis of three-dimensional flexible riser structures. *Ocean Engineering, 38*(5/6), 793-803.

Lacasse, S. (2001). *Geotechnical solutions for the offshore synergy of research and practice*. Houston, TX: University of Houston.

Mullapudi, R., & Ayoub, A. (2010). Nonlinear finite element modeling of beams on two-parameter foundations. *Computers and Geotechnics, 37*(3), 334-342.

Nedjar, D., Hamane, M., Bensafi, M., Elachachi, S., & Breysse, D. (2007). Seismic response analysis of pipes by a probabilistic approach. *Soil Dynamics and Earthquake Engineering, 27*(2), 111-115.

Phoon, K.-K., & Kulhawy, F. H. (1999). Evaluation of geotechnical property variability. *Canadian Geotechnical Journal, 36*(4), 625-639.

Popescu, R., Deodatis, G., & Nobahar, A. (2005). Effects of random heterogeneity of soil properties on bearing capacity. *Probabilistic Engineering Mechanics, 20*(4), 324-341.

Reddy, J. N. (2015). *An introduction to nonlinear finite element analysis: With applications to heat transfer, fluid mechanics, and solid mechanics*. Oxford, UK: Oxford University Press.

Santos, H. A. F. A. (2015). A novel updated Lagrangian complementary energy-based formulation for the elastica problem: Force-based finite element model. *Acta Mechanica, 226*(4), 1133-1151.

Sapountzakis, E., & Kampitsis, A. E. (2011). Nonlinear analysis of shear deformable beam-columns partially supported on tensionless three-parameter foundation. *Archive of Applied Mechanics, 81*, 1833-1851.

Sapountzakis, E., & Kampitsis, A. E. (2013). Inelastic analysis of beams on two-parameter tensionless elastoplastic foundation. *Engineering Structures 48,* 389-401.

Seguini, M., & Nedjar, D. (2017a). Modelling of soil-structure interaction behaviour: geometric nonlinearity of buried structures combined to spatial variability of soil. *European Journal of Environmental Civil Engineering, 21*(10), 1217-1236.

Seguini, M., & Nedjar, D. (2017b). Nonlinear analysis of deep beam resting on linear and nonlinear random soil. *Arabian Journal for Science Engineering*, *42*, 3875-3893.

Seguini, M., & Nedjar, D. (2020). Dynamic and probabilistic analysis of shear deformable pipeline resting on two parameter foundation. *Periodica Polytechnica Civil Engineering, 64*(2), 430-437.

Seguini, M., Khatir, S., Nedjar, D., & Wahab, M. A. (2022). *Machine learning for predicting pipeline displacements based on soil rigidity.* Paper presented at the Proceedings of the 10th International Conference on Fracture Fatigue and Wear: FFW 2022, 2-3 August, Ghent University, Belgium.

VanMarcke, E. H. (1983). *Random fields: Analysis and synthesis*. Cambridge MA: MIT Press.