2D continuous wavelet transform for pattern recognition: Application to the Azadi tower

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ABSTRACT

This paper explores the unique capabilities of the Two-Dimensional Continuous Wavelet Transform (2D-CWT) in Structural Pattern Recognition (SPR). It uses the historic Azadi Tower as a case study due to its diverse structural patterns across various scales and angles. Thus, the novelty of the work is to apply the 2D-CWT for pattern recognition of a historic structure. It examines the effects of scale and angle indices on target pattern extraction and the influence of different wavelet functions on recognition outcomes. Results show that reducing the angle index enhances edge detection, allowing for identifying edges and patterns from multiple directions. Moreover, higher scales of 2D-CWT are sensitive to local abrupt changes in images, underscoring the transform's dual capabilities. The paper also highlights 2D-CWT's proficiency in detecting directional edges and singularities.

1. Introduction

Pattern recognition methods have become essential for Structural Health Monitoring (SHM) in identifying structural changes, such as damage. Typically, these applications use outlier analysis of damage-sensitive features derived from SHM data to detect such changes Sohn et al. (2001). Due to the potential existence of different structural patterns in structures and different pattern recognition goals, other geometries can be recognized in structures.

Edge detection is vital for pattern recognition, and extensive research has been conducted in this field. For example, Wei et al. (2004) developed a novel algorithm for automatically extracting building footprints from high-resolution QuickBird satellite images, shifting from traditional aerial photography to more cost-effective satellite imagery. The findings confirmed the proposed algorithm's effectiveness in accurately identifying and delineating building outlines in urban areas. The method demonstrated accuracy and efficiency by integrating unsupervised clustering, shadow detection, and edge detection (using the Canny operator and Hough transform). A comparison with manually delineated boundaries showed it to be a viable alternative for extracting buildings from satellite imagery. Saadatmorad, Rezaei-Sedehi, et al. (2024) developed an effective method for detecting cracks in historical structures, specifically focusing on the Si-o-se-pol Bridge in Iran. The goal was to create a faster and more accurate damage detection tool to aid preservation efforts. They modified the Prewitt filter for crack detection. The study demonstrated that this new, improved filter performed significantly better in identifying cracks than the standard vertical and horizontal Prewitt filters. This suggests the modified filter is more effective for assessing damage in historical structures. Avdelidis and

Moropoulou (2004) employed thermal images of historical building surfaces to determine their conditions, indicating that infrared thermography is an effective method for evaluating the materials of historic structures to safeguard cultural heritage.

Wang et al. (2018) employed convolutional neural networks to differentiate healthy from damaged bricks, while Germanese et al. (2018) used UAVs and planar markers to monitor cracks in historic buildings. The importance of image processing for damage detection has grown with continuous improvements in digital image quality from advanced cameras. Yang et al. (2011) suggested an enhanced Prewitt algorithm for edge detection in sample images, citing the traditional Prewitt algorithm's sensitivity to noise as a key motivation for their improvement. Abdel-Qader et al. (2003) employed four edge detection methods - Fast Haar Transform, Canny, Fast Fourier Transform, and Sobel - to identify edges in concrete bridges, with Fast Haar Transform yielding the best outcomes. Ahmed (2018) compared three edge detection algorithms - Sobel, Prewitt, and Canny - implemented in C# on various images. The goal was to extract edge information crucial for object recognition, shape analysis, and texture identification. Performance was evaluated based on processing time and accuracy, using Mean Squared Error and Peak to Signal Noise Ratio. Results indicated that the Canny algorithm outperforms the others.

This paper explores the effectiveness of the 2D continuous wavelet transform (2D-CWT) for pattern recognition, using the Azadi Tower, a notable example of Iranian-Islamic architecture in Tehran, as a case study.

2. Methodology

By definition, the one-dimensional continuous wavelet transform (1D-CWT) is expressed as follows (Abid et al., 2008):

$$1D - CWT(s,t) = \frac{1}{s}\int_{-\infty}^{+\infty} f(x)\ \psi^{*}\left(\frac{x-t}{s}\right) dx \tag{1}$$

Where s and t are the scaling and shifting parameters, respectively. $f(x)$ is the original signal. $\psi^{*}(x)$ is the complex form of a wavelet function, and $\psi\left(\frac{x-t}{s}\right)$ shows the scaled and translated version of the wavelet function $\psi(x)$. A wavelet function satisfies the following conditions (Saadatmorad, Khatir, et al., 2024):

$$\int_{-\infty}^{+\infty} \psi(x) dx = 0 \tag{2}$$

$$\int_{-\infty}^{+\infty} \psi(x) dx = 0 \tag{3}$$

$$\int_{-\infty}^{+\infty} |\psi(x)|^{2} dx < \infty \tag{4}$$

Some wavelet functions have exact formulas, while others are represented numerically. For instance, the Mexican hat wavelet function is expressed as follows:

$$f(x) = e^{-x^{2}} \tag{5}$$

The parametric formulation of the Gaussian function is expressed as follows:

$$f(x) = ae^{(-\frac{(x-b)^{2}}{2c^{2}})} \tag{6}$$

A and b are fundamental constants, and c is a non-zero constant. Geometrically, the Gaussian function is a function. In addition, c is the standard deviation and controls the width of the Gaussian curve.

Or in a standard form:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \tag{7}$$

The parameter μ is called the mean of the distribution. Likewise, σ is called the standard deviation, and the variance of the distribution is represented by σ^2.

By adjusting the parameters and differentiating the function twice, we formulate the Ricker wavelet (or Mexican hat wavelet) as follows (Singh et al., 2022):

$$\psi(x) = \frac{2}{\sqrt{3\sigma}\sigma\pi^{\frac{1}{4}}}(1-(\frac{x}{\sigma})^2)e^{\frac{-x^2}{2\sigma^2}} \tag{8}$$

Therefore, 1D-CWT for the Mexican hat wavelet is rewritten as follows:

$$1D - CWT(s,t) = \frac{1}{s}\int_{-\infty}^{+\infty} f(x)\ \frac{2}{\sqrt{3\sigma}\sigma\pi^{\frac{1}{4}}}(1-(\frac{\left(\frac{x-t}{s}\right)}{\sigma})^2)e^{\frac{-\left(\frac{x-t}{s}\right)^2}{2\sigma^2}}dx \tag{9}$$

Also, 2D-CWT can be defined as follows:

$$2D - CWT(s,t,u) = \frac{1}{s}\int_{-\infty}^{+\infty} f(x,y)\ \psi^*\left(\frac{x-t}{s},\frac{y-u}{s}\right)dxdy \tag{10}$$

The 2D-Mexican Hat wavelet function is utilized in the two-dimensional wavelet transform, as expressed below:

$$\psi(x,y) = \frac{1}{\pi}(1-\frac{x+y}{2})e^{\frac{-(x^2+y^2)}{2}} \tag{11}$$

Hence, 2D-CWT for the Mexican hat wavelet is expressed as follows:

$$2D - CWT(s,t,u) = \frac{1}{s}\int_{-\infty}^{+\infty} f(x,y)\frac{1}{\pi}(1-\frac{\frac{x-t}{s}+\frac{y-u}{s}}{2})e^{\frac{-((\frac{x-t}{s})^2+(\frac{y-u}{s})^2)}{2}}dxdy \tag{12}$$

Where $f(x,y)$ is a two-dimensional function, and $2D - CWT(s,t)$ is the wavelet coefficients obtained from the two-dimensional continuous wavelet transform.

This is the scalar representation of the 2D-CWT, while the vector representation is used in practice. We define the mother wavelet $\psi(\vec{X})$. For this mother wavelet, the following condition has to be satisfied (Saleh et al., 2022):

$$\psi(\vec{0}) = 0 \leftrightarrow \int \psi(\vec{X})\, d^2\vec{X} = 0,\quad \vec{X} \in R^2 \tag{13}$$

The shifted and scaled version of this wavelet function is expressed as follows:

$$\psi_{a,\theta,\vec{b}}(\vec{X}) = \frac{\psi\left[\mathcal{R}_\theta^{-1}\left(\frac{\vec{X}-\vec{b}}{a}\right)\right]}{a} \qquad a \in R^+\ , \qquad \vec{b} \in R^2 \tag{14}$$

The equation above shows that the surface integral of $\psi(\vec{X})$ has a zero mean, enabling the detection of potential local singularities in the signal after convolution. Hence, the directional 2D-CWT is defined as follows:

$$\psi_{a,\theta,\vec{b}}(\vec{X}) = \frac{\psi\left[\mathcal{R}_{\theta}^{-1}\left(\frac{\vec{X}-\vec{b}}{a}\right)\right]}{a} \qquad a \in R^{+}\ ,\ \vec{b} \in R^{2} \tag{15}$$

Where:

$$r_{\theta} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{16}$$

And for the directional Mexican hat wavelet (proposed in 1980 by Marr & Hildreth, 1980), we have (Wang & Lu, 2010):

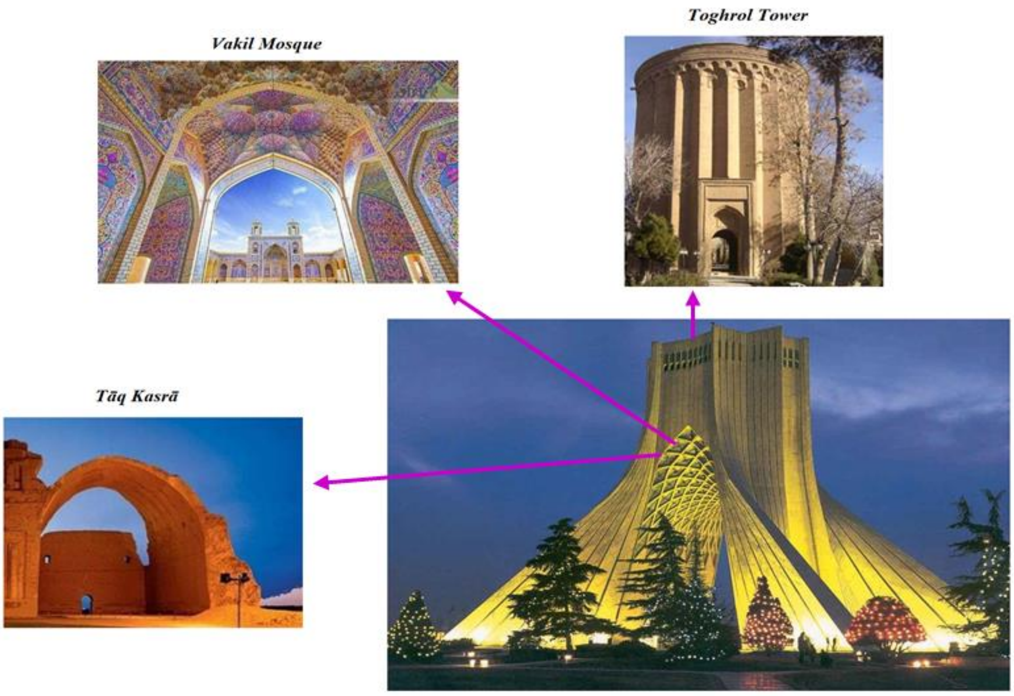
$$\psi(\vec{X}) = (2 - |\vec{X}|^{2})e^{-\frac{|\vec{X}|^{2}}{2}} \tag{17}$$

3. Azadi Tower

The Azadi Tower, previously known as Shahyad Tower, is in Azadi Square, Tehran, Iran. Designed by architect Hossein Amanat, it was built in 1971 and is regarded as one of Tehran’s most significant landmarks (Diba, 2012). At 21, Hossein Amanat designed the tower, drawing considerable influence from his extensive travels across Iran and exposure to diverse architectural styles. Figure 1 shows the sources of inspiration he referenced.

Figure 1

The Sources of Inspiration in the Designing Process



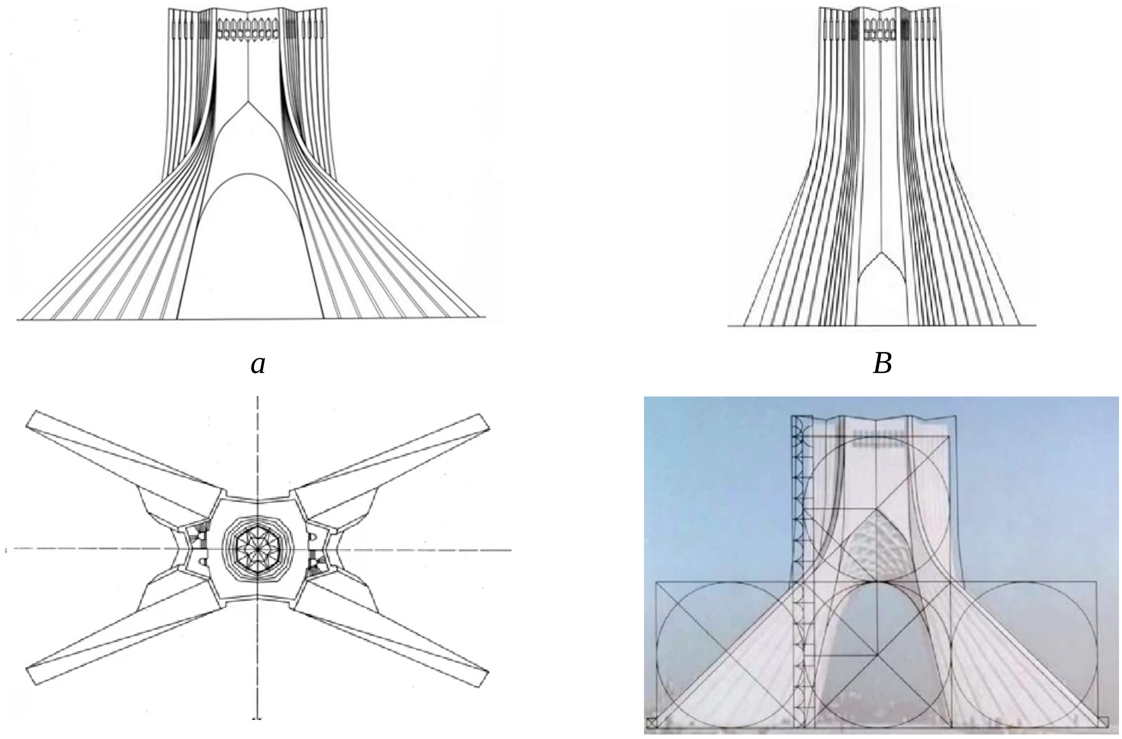
Source. Created by Author

Hossein Amanat attributes his success in this work to traveling and seeing various works of Iranian architecture. He believes that the various Iranian-Islamic architectural buildings he visited during his student camps and scientific trips caused the elements of the Azadi Tower to form in his mind unconsciously. He says: “When people gather around the Azadi Tower, this tower is like a kind father with warm and open arms, wanting to embrace them." (https://www.aparat.com/v/k37e6me).

Amanat describes this building as a blend of pre-Islamic and post-Islamic Iranian architecture, with the plans illustrated in Figure 2 and geometric proportions highlighted in Figure 2d.

Figure 2

Azadi Tower's Plans: A) East-West View, B) South-North View, C) Top View, and D) Some Geometric Proportions



a *B*

c *D*

Source. https://unsplash.com

4. Results

This section examines the use of 2D-CWT for pattern recognition, focusing on edge and singularity detection. The Azadi Tower case study is chosen for its various directional edges and singularities. Table 1 presents the scale and angle index parameters employed in this research for 2D-CWT, which are applied in different pattern detection scenarios.

Table 1

The Scale Index and Angle Index Parameters Used in This Research for The 2D-CWT

N	Scale index (a)	Angle index ($\boldsymbol{\theta}$)	
		Radians	Degrees
1	1	0	0
2	2	$\frac{\pi}{4}$	45°
3	3	$\frac{\pi}{2}$	90°

N	Scale index (a)	Angle index (θ)	
4	4	$\frac{3\pi}{4}$	135°
5	5	π	180°
6	6	$\frac{5\pi}{4}$	225°
7	7	$\frac{3\pi}{2}$	270°
8	8	$\frac{7\pi}{4}$	315°

Source. Created by author

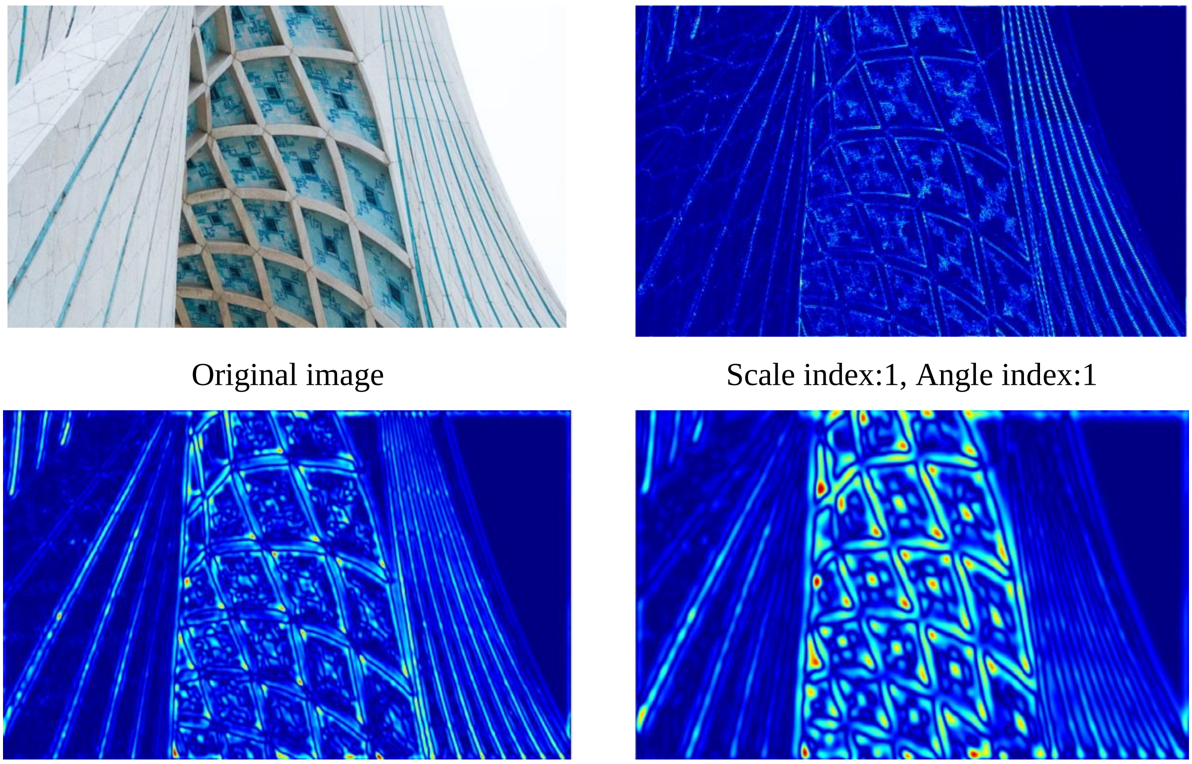
The impact of increasing the scale index, angle index, shading, and wavelet function type is analyzed.

4.1. The effect of increasing scale index

This section considers the effect of increasing the scale index on the results of 2D-CWT. The Mexican hat wavelet function is used to analyze an original image of the Azadi Tower. Figure 3 indicates the effect of increasing the scale index on pattern recognition outputs. As shown, at a fixed angle index (here 1), as the scale increases, it becomes harder to identify the fine wavelet edges in the image; however, it becomes easier to identify the singularity.

Figure 3

The Effect of Increasing Scale Index on The Results of 2D-CWT



Original image

Scale index:1, Angle index:1

Scale index:2, Angle index:1

Scale index:3, Angle index:1

Source. https://unsplash.com

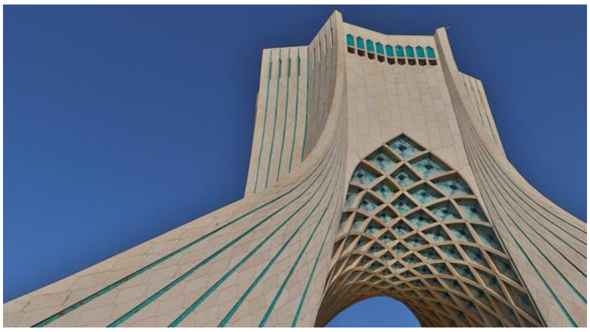
Thus, the lowest scale is best for edge detection, while higher scales enhance singularity detection using the continuous 2D wavelet transform. Therefore, this transform effectively performs two tasks: edge detection at lower scales and singularity detection at higher scales.

4.2. The effect of increasing the angle index

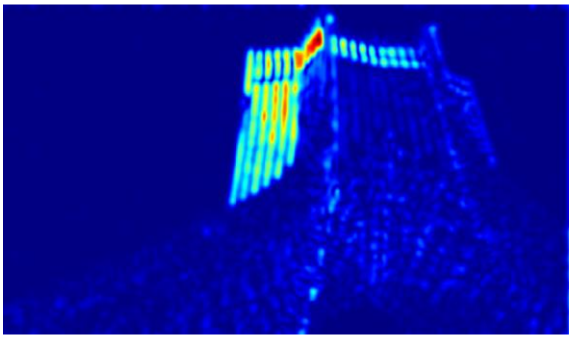
This section deals with the effect of increasing the angle index on the results of 2D-CWT. The Mexican hat wavelet function is used to analyze an original image of the Azadi Tower. As shown in Figure 4, to evaluate the effect of increasing the angle index on the results of 2D-CWT, we fix the scale index equal to 2 and increase the angle index. Figure 4 shows that different edges are identified in the structure according to the angle index. In other words, according to Table 1 and Figure 4, edges perpendicular to the selected angle are highlighted.

Figure 4

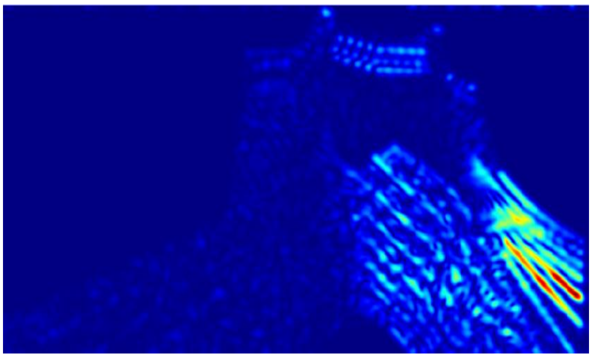
The Effect of Increasing Angle Index on The Results of 2D-CWT



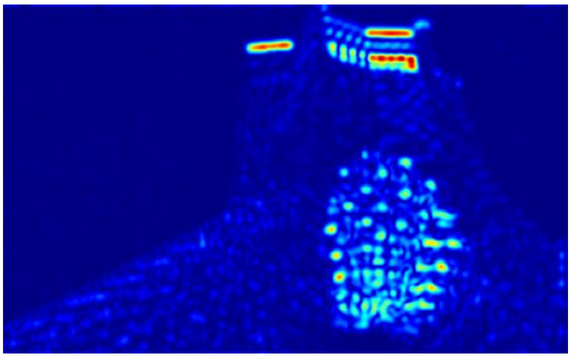
Original image



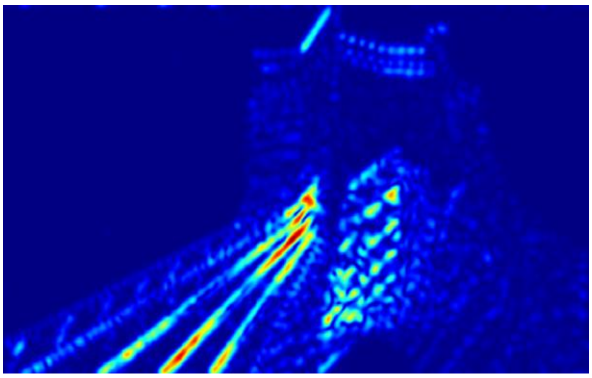
Scale index:2, Angle index:1



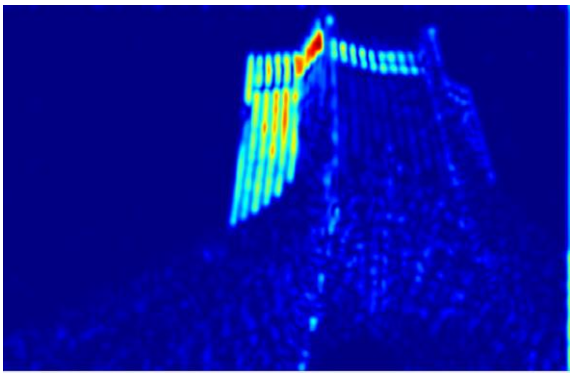
Scale index:2, Angle index:2



Scale index:2, Angle index:3



Scale index:2, Angle index:4



Scale index:2, Angle index:5

Source. https://unsplash.com

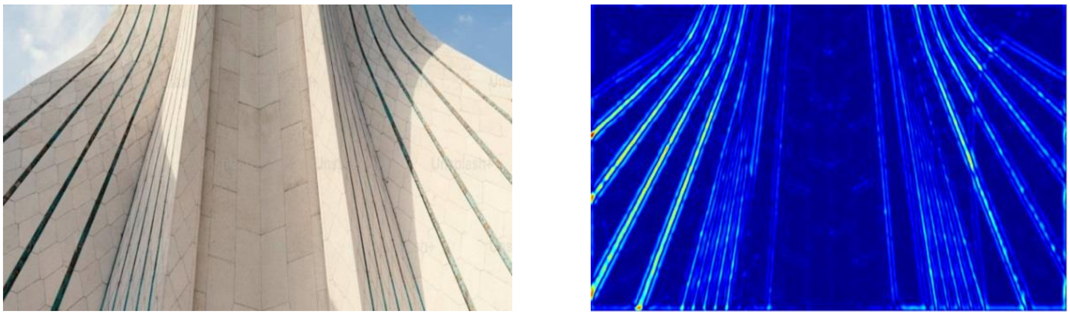
One weakness of edge detection algorithms is that they simultaneously highlight the edges of an image or identify them in two horizontal and vertical directions. The discrete 2D wavelet transform performs this separation in three vertical, horizontal, and diagonal directions, but it is a more suitable tool for singularity detection than edge detection. The results of this research show that the continuous 2D wavelet transform can detect edges and structural patterns in different directions and, at the same time, by adjusting the scale and angle correctly, it can also be a suitable method for singularity detection in the image.

4.3. The effect of shade

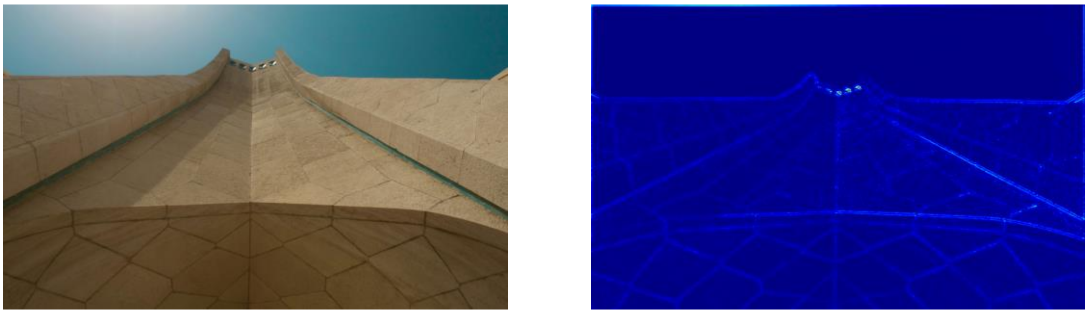
In image processing, one factor that hinders the correct processing of patterns is the presence of shadows in the original image. This section investigates the effect of shadows on the accuracy of pattern recognition by 2D-CWT.

Figure 5

The Effect of Shadow on Performance of 2D-CWT



Original image Scale index:2, Angle index:1



Original image Scale index:2 , Angle index:1

Source. https://unsplash.com

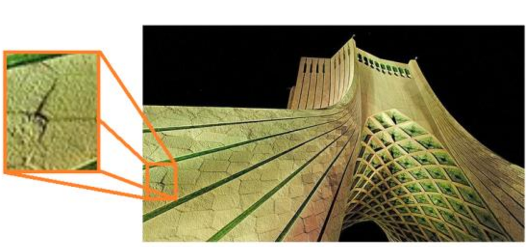
Figure 5 shows two images of the Freedom Tower with two different shadow intensities. In the first case, the shadow is more intense, creating a higher color contrast with the un-shadowed part of the image. Meanwhile, in the second image, the shadow intensity is less intense, creating a lower color contrast with the un-shadowed part of the structure. Hence, images with shadows yield varied edge detection results. While shadows intensify contrast, their impact on edge detection is even more significant.

4.4. Singularity detection by 2D-CWT

This section demonstrates that 2D-CWT identifies singularities in addition to edges. Figure 6 illustrates the color singularity in the original image of the Freedom Tower. The results indicate that at a fixed angle index, singularity detection improves with an increasing scale index. Specifically, lower scales reveal local and smaller edges, while higher scales highlight the most prominent singularities in the image.

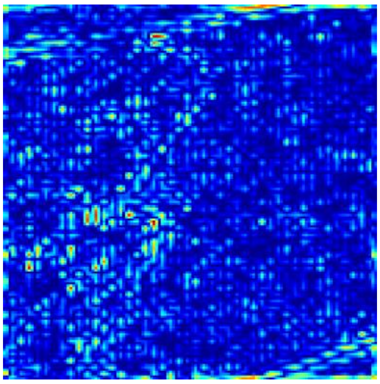
Figure 6

Results of Singularity Detection by 2D-CWT

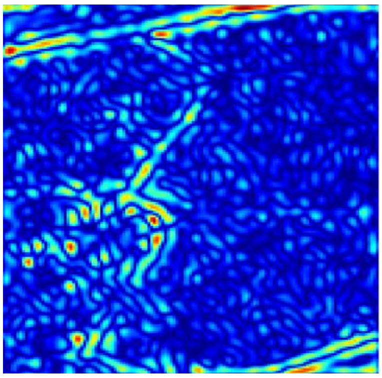


A singularity in the building

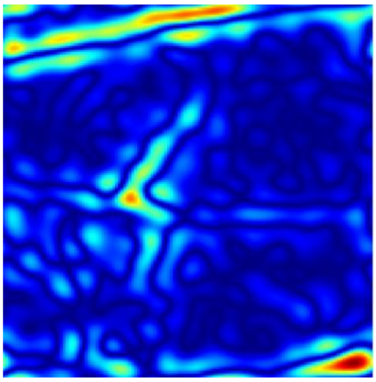
Original image



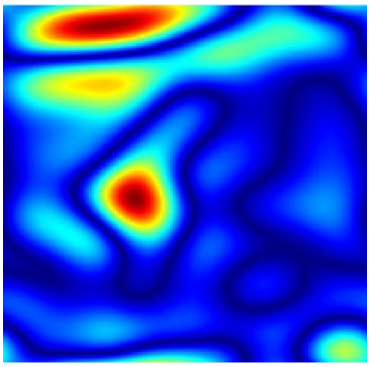
Scale index:1, Angle index:1



Scale index:2, Angle index:1



Scale index:5, Angle index:1



Scale index:14, Angle index:1

Source. Created by author

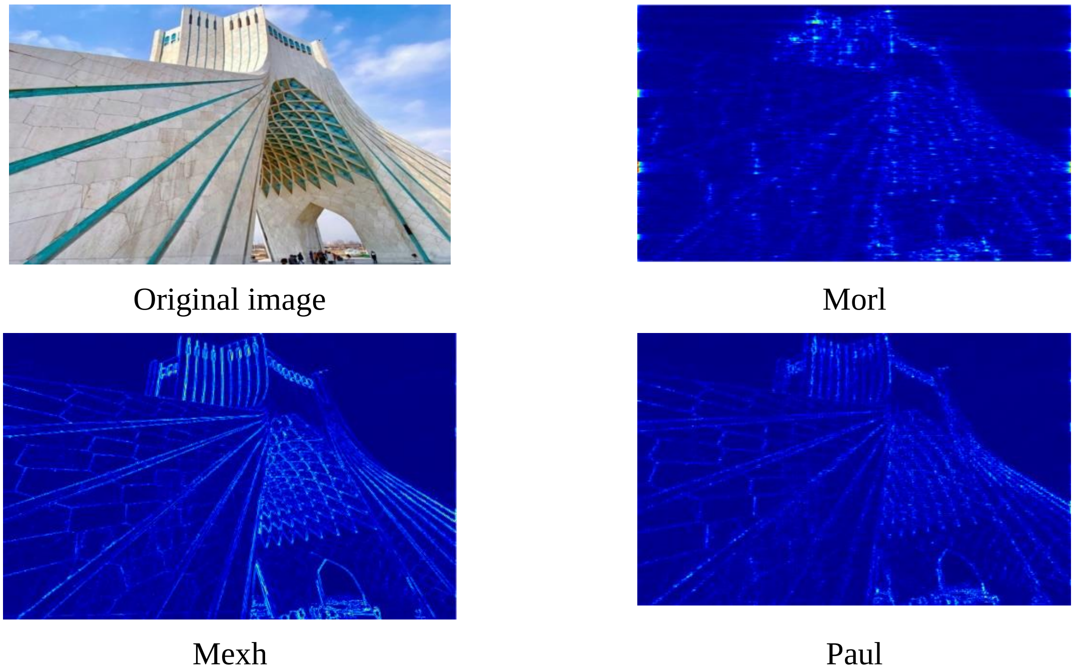
The results in Figure 6 indicate that as scale increases at a fixed angle, the global singularities of the image become more prominent, while lower scales emphasize smaller singularities (edges). Thus, the objectives of structural pattern recognition using 2D-CWT significantly influence its application and the selection of angle and scale parameters.

4.5. The effect of the type of wavelet function

This section examines how various wavelet functions affect pattern recognition results using 2D-CWT. Specifically, Morlet wavelet function (Morl), Mexican hat wavelet (Mexh), and Paul wavelet function are employed. The impact of varying the scale and angle indices on the 2D-CWT outcomes is also assessed.

Figure 7

Result of 2D-CWT for Three Different Wavelet Functions: Scale Index:1, Angle Index:1

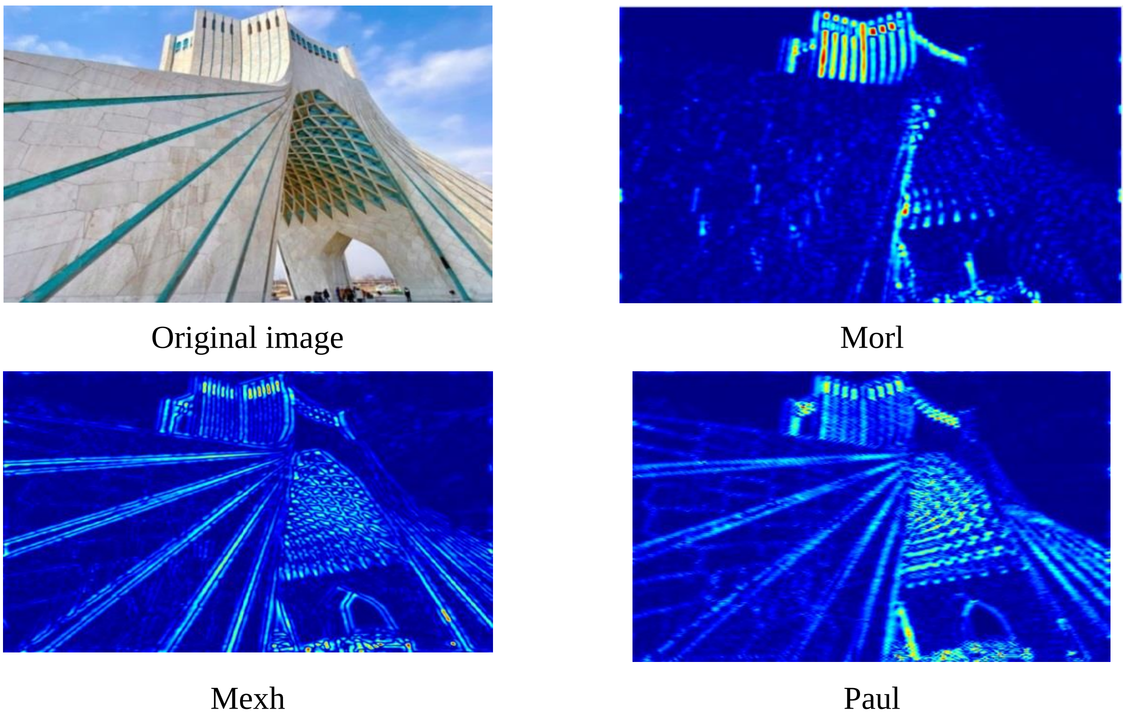


Source. Created by author

Figure 7 shows that different wavelet functions at a fixed scale and angle may yield different results. Even with equal angle and scale indices, some wavelet functions may produce different results due to their geometric shape.

Figure 8

Result of 2D-CWT for Three Different Wavelet Functions: Scale Index:2, Index of Angle:1



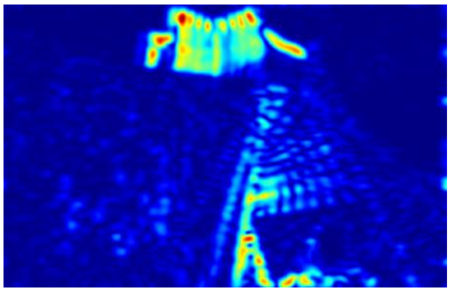
Source. Created by author

Figures 7 and 8 show that Mexh is the best wavelet function among the three edge detection functions for edge detection.

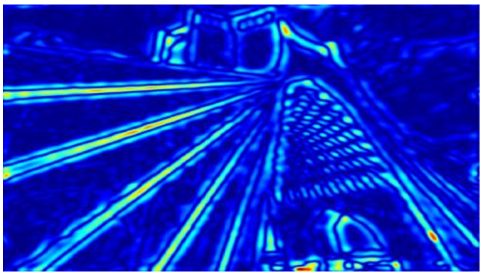
Figure 9

Result of 2D-CWT for Three Different Wavelet Functions: Scale Index:3, Angle Index:1

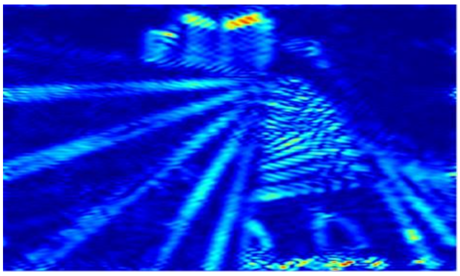
Original image



Morl



Mexh



Paul

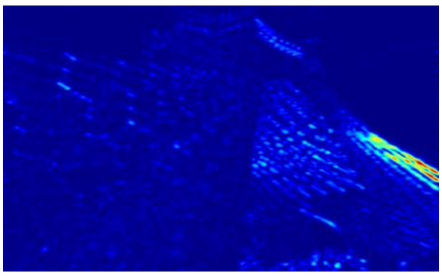
Source. Created by author

Figures 9 and 10 show that as the scale increases, the brightness intensity of the image's global edges increases, with 2D-CWT focusing on detecting global patterns. Conversely, 2D-CWT targets local patterns in the image as the scale decreases.

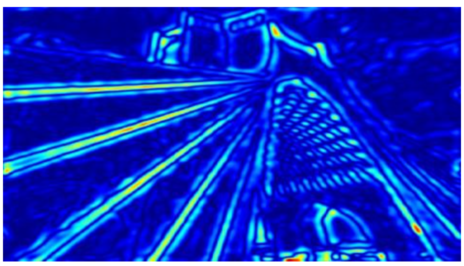
Figure 10

Result of 2D-CWT for Three Different Wavelet Functions: Scale Index:2, Angle Index:2

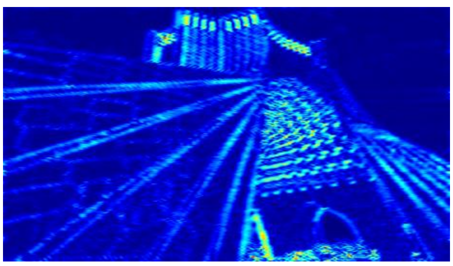
Original image



Morl



Mexh



Paul

Source. Created by author

5. Conclusions

A proper understanding of the wavelet transform enables practical structural analysis. This research investigates how the continuous two-dimensional wavelet transform identifies structural patterns. The findings highlight that a key benefit of 2D-CWT is its ability to detect and separate edges in multiple directions, distinguishing it from other transforms. At higher scales, 2D-CWT effectively identifies singularities alongside edges. Results indicate that edge detection is more efficient at lower scales, while singularity detection is more effective at higher scales.

Additionally, the angle parameter can be adjusted for targeted edge detection. Also, the results show that the degree of recognition of a shadow as an edge depends on the contrast intensity that the shadow has in the object or structure. As the contrast of the shadow increases relative to the rest of the image, the shadow boundary is recognized as an edge. Therefore, to correctly identify the pattern in the structure, one should not process the photos with a high-contrast shadow.

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